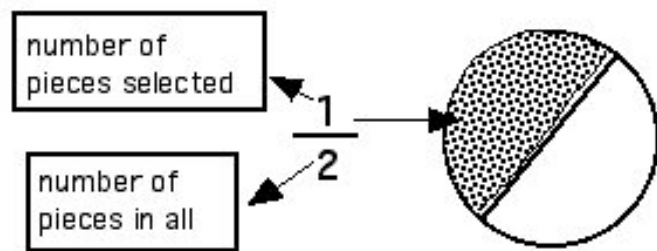


## Three meanings of fraction

There are three meanings of fraction that are relevant to the early years mathematics curriculum. [There are two more fraction meanings (ratio and indicated division) that are relevant to the middle years mathematics curriculum.]

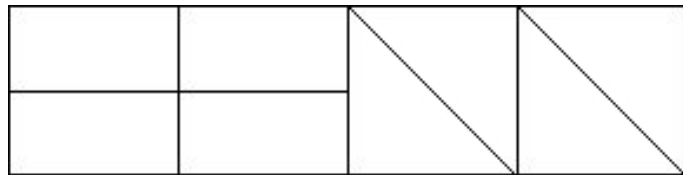
### 1. The part of a whole meaning

When a 5-year old child says “I ate half of the cookie.” he/she is expressing a part-whole relationship. The child uses ‘half’ not in the sense of a number but in the sense of an actual or imagined action that involves cutting a whole physical object in the middle. The imagined or actual action of cutting a whole object into ‘n’ equivalent/equal parts underlies the cut meaning of fraction. We represent each part symbolically by the fraction notation ‘1/n’. The circle diagram here indicates this.



‘Equivalent/equal’ means equivalent/equal according to a measurement attribute such as length, area, volume, etc. In the case above, the two pieces of the circle are equal in area and they happen to look the same. That does not need to be the case.

Consider a granola bar cut in the way shown in the diagram. The 8 pieces do not all look the same. Yet each piece is  $\frac{1}{8}$  of the granola bar because the pieces have the same area.



In summary, the part of a whole meaning of fraction involves cutting a naturally existing whole into equal parts according to measurable qualities such as length, area, volume, mass, etc. An example for length could be a string cut into 4 parts of equal length. Each part is  $\frac{1}{4}$  of the length of the whole string. An example for area is the granola bar or circle example above. An example for volume could be a loaf of bread cut into 8 parts having the same volume (a difficult thing to actually do). Each part is  $\frac{1}{8}$  of the volume of the whole loaf.

## 2. The part of a group or set meaning

The part of a group/set meaning does not involve cutting a whole into equal parts. Nor does it involve a natural whole. The part of a group meaning involves selecting objects from a group on some basis. A group is not a natural whole as is a pie (for example). Unlike the part of a whole meaning, the part of a group/set meaning does not require that the objects be of the same size or type. Neither is there any cutting involved.

Consider the shapes shown in the diagram. They are not all equal in area and some have different orientations. One can say though, for example, that  $\frac{4}{9}$  of the shapes are curved.

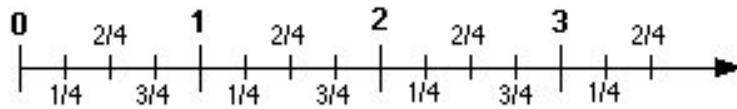


Another example. Suppose there are 23 books of varying size and content on a shelf where 14 of them are novels. We can represent this situation by the fraction  $\frac{14}{23}$ . In this case, we mean by  $\frac{14}{23}$  that 14 out of the 23 books are novels.

The part of a group/set meaning is significantly different from the part of the whole meaning. The part of a group/set meaning involves mentally placing discrete things into categories (e. g. red, prime), a different enterprise than cutting up things according to a measurement attribute such as length, area, volume, mass, . . .

### 3. The name for a point (or measure) meaning

The name for a point meaning involves associating marks on measuring devices such as rulers with fraction names. The part of a whole meaning underlies this



meaning as the marks are obtained by cutting a section of a line into equal parts. Notice in the number line here that there are a lot of  $1/4$  lengths (for example). Yet each is different because they each name a different point on the number line (e. g.  $1/4$ ,  $1 \frac{1}{4}$ ,  $2 \frac{1}{4}$ , etc.).

The name for a point meaning is the most abstract of the first three meanings of fraction, and, overall, the most useful for purposes of teaching fraction computational skills.

A number line that contains fraction-named points can serve as a ruler for measuring length using fractional amounts of units of length. For this reason, some refer to the name for a point meaning as the measure meaning of fraction.